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ATMOSPHERIC MODELS FOR USE IN HYDRODYNAMIC COMPUTER CODES

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Lieutenant USAF

Edmund A. Nawrocki
Captain USAF

William A. Whitaker
Major USAF

TECHNICAL REPORT NO. AFWL-TR-67-75

August 1967

AIR FORCE WEAPONS LABORATORY
Research and Technology Division
Air Force Systems Command
Kirtland Air Force Base
New Mexico

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FOREWORD

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Inclusive dates of research were 1 August 1964 to 1 September 1964. The report was submitted 24 July 1967 by the Air Force Weapons Laboratory Project Officer, Lt Jeremiah U. Brackbill (WLRT).

This technical report has been reviewed and is approved.

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ABSTRACT

(Distribution Limitation Statement No. 2)

A hydrostatically stable atmospheric model is necessary to perform theoretical calculations of hydrodynamic motion in the atmosphere, on a digital computer. This report presents three such models developed at the Air Force Weapons Laboratory for use in its hydrodynamic computer codes. One is for the annual mean temperate atmosphere (45° N latitude); one for the annual mean tropical atmosphere (15° N latitude); and one for the summer subarctic atmosphere (60° N latitude). The models are presented herein in tabular form and as FORTRAN subroutines which could be placed directly into any hydrodynamic computer code. For a given altitude (cm), the subroutines return a pressure (dynes/cm^2), density (gms/cm^3), specific internal energy (ergs/gm), temperature (°K), and (γ - 1). The pressures and densities agree with tabulated values to at least 1 part in 10^8 and temperatures to at least 1 part in 100. The atmospheres experience an acceleration of no more than 3 parts in 10^3 in a first order finite difference scheme with a zone size of 1 kilometer, the worst case.

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TABLES

SECTION I

INTRODUCTION

The Air Force Weapons Laboratory is engaged in research on the phenomenology of atmospheric nuclear weapons detonations. This research consists of numerical experiments--theoretical calculations of various yield-altitude detonations--performed on large memory, high-speed computers. These calculations are made by a series of large, complex, computer codes that enable prediction, from essentially first principles, of the phenomenology from microseconds to minutes after a detonation.

Three main codes are used: SPUTTER, a one-dimensional radiation transport Lagrangian hydrodynamic code; SAP, a one-dimensional Lagrangian hydrodynamic code; and SHELL, a two-dimensional Eulerian hydrodynamic code that also has the capability of doing radiation diffusion. The first, SPUTTER, takes the radiative output directly from weapons design calculations, deposits the energy in air, and calculates the radiative and hydrodynamic growth of the fireball to a time of about 1 second. It uses a multifrequency transport scheme involving typically 20 frequency groups. The output of this code is used as input to SAP, which calculates the hydrodynamic expansion of the shock wave at various angles to the horizontal. The output of SPUTTER is also used as input to SHELL, which calculates fireball growth and rise to late times.

These codes, or any hydrodynamic code in general, which perform hydrodynamic calculations in a real atmosphere, need an atmospheric model that is stable in a R^{-2} gravity field; otherwise, the numerical integration of the hydrodynamic equations will result in fictional vertical velocity components.

This report describes three stable atmospheric models developed at the Air Force Weapons Laboratory. The first model is based on data in reference 1 for an annual mean, temperate atmosphere (45° N latitude) for altitudes from sea level to 700 kilometers. The second is based on data in reference 2 for an annual mean, tropical atmosphere (15° N latitude) for altitudes from sea level

to 90 kilometers and reference 1 for altitudes from 90 to 700 kilometers. The third is based on data in reference 3 for a summer subarctic atmosphere (60° N latitude) for altitudes from sea level to 90 kilometers and reference 1 for altitudes from 90 to 700 kilometers

The Appendix contains a FORTRAN subroutine for the models.

SECTION II

DERIVATION OF THE MODELS

1. Conditions and Assumptions

The molecular scale temperature is the defining property of the US Standard Atmosphere, 1962 (Ref. 1). It is defined by the relationship

$$T_m(z) = \frac{M_o}{M(z)} T(z) \qquad (1)$$

where $T_m(z)$ ≡ molecular scale temperature at altitude z

$T(z)$ ≡ absolute temperature at altitude z

$M(z)$ ≡ molecular weight of air at altitude z

M_o ≡ molecular weight of air at sea level

The variation of the molecular scale temperature is defined as a series of connected segments linear in geopotential altiude to 90 kilometers and in geometric altitude above 90 kilometers (Ref. 1). Therefore, we divide the temperate, tropical, and subarctic models into 21, 22, and 23 altitude groups respectively from sea level to 700 kilometers and assume a linear molecular scale temperature variation with altitude within each altitude group. This variation has the form

$$T_m(z) = T_m\left(z_i\right) + \left(L_i\right)\left(z-z_i\right) \qquad (2)$$

where z_i ≡ the base altitude of the i-th altitude group

L_i ≡ the molecular scale temperature gradient over the i-th altitude group

$$= \frac{T_m\left(z_{i+1}\right) - T_m\left(z_i\right)}{z_{i+1} - z_i}$$

We also assume the atmosphere behaves like an ideal gas in which case

$$\rho(z) = \frac{M(z)p(z)}{RT(z)} \tag{3}$$

where $\rho(z)$ ≡ the density at altitude z

$p(z)$ ≡ the pressure at altitude z

R ≡ the universal gas constant

Finally, we require the atmospheric models to satisfy the condition of stability given by the hydrostatic equation

$$\frac{d[p(z)]}{dz} = -\rho(z)g(z) \tag{4}$$

where $g(z)$ ≡ the acceleration due to gravity at altitude z

$$= \frac{g_o a^2}{(a+z)^2} \tag{5}$$

g_o ≡ the acceleration due to gravity at sea level

a ≡ the radius of the earth

By combining equations 1 through 4, the hydrostatic equation becomes

$$\frac{d[p(z)]}{p(z)} = -\frac{g_o M_o a^2}{R} \frac{dz}{(a+z)^2\left[T_m\left(z_i\right) + \left(L_i\right)\left(z-z_i\right)\right]} \tag{6}$$

Integration of equation 6 from z_i to z results in the final form, which is used to calculate pressures in each altitude group.

$$\ln\frac{p(z)}{p\left(z_i\right)} = -\frac{g_o M_o a^2}{\left[T_m\left(z_i\right) - \left(L_i\right)\left(a+z_i\right)\right]^2 R} \left\{ \frac{\left[T_m\left(z_i\right) - \left(L_i\right)\left(a+z_i\right)\right]\left(z-z_i\right)}{(a+z)\left(a+z_i\right)} + L_i \ln\left[\frac{a+z_i}{(a+z)} \frac{T_m\left(z_i\right) + \left(L_i\right)\left(z-z_i\right)}{T_m\left(z_i\right)}\right]\right\} \tag{7}$$

2. Calculations

To calculate the molecular scale temperatures, $T_m(z_i)$, and the molecular scale temperature gradients, L_i, which define each atmospheric model, we take the pressures listed in references 1, 2, and 3 corresponding to the base altitude of each altitude group and the molecular scale temperatures as defined by equation 2 to be constants. With $p(z_{i+1})$, $p(z_i)$, $T_m(z_i)$, a, g_o, R, and M_o known, equation 7 becomes

$$\ln \frac{p(z_{i+1})}{p(z_i)} = -\frac{g_o M_o a^2}{\left[T_m(z_i) - (L_i)(a+z_i)\right]^2 R} \left\{ \frac{\left[T_m(z_i) - (L_i)(a+z_i)\right](z_{i+1}-z_i)}{(a+z_{i+1})(a+z_i)} + L_i \ln\left[\frac{a+z_i}{a+z_{i+1}} \; \frac{T_m(z_i) + (L_i)(z_{i+1}-z_i)}{T_m(z_i)}\right] \right\} \quad (8)$$

which is solved by an appropriate iterative technique to find that molecular scale temperature gradient, L_i, which will satisfy equation 8 and thus, satisfy the condition of hydrostatic stability for the i-th group. Then the molecular scale temperature corresponding to the base altitude of the next altitude group becomes

$$T_m(z_{i+1}) = T_m(z_i) + (L_i)(z_{i+1}-z_i) \quad (9)$$

where L_i is the gradient just calculated. Then the calculation proceeds to the next altitude group where the exact procedure is repeated. This procedure is repeated until the molecular scale temperature and gradient have been calculated for all altitude groups.

The calculation begins at sea level for which we take the molecular scale temperature as listed in references 1, 2, and 3. (This temperature can be thought of as an "initial condition." All other temperatures are defined by the gradients obtained from the solution of equation 8 and equation 9.) Solution of equation 8 provides L_1 which is used in equation 9 to obtain T_2. Using T_2, equation 8 gives L_2, which in turn by equation 9 gives T_3, and so forth.

We repeat the above iteration for each model using the appropriate data for $p(z_1)$, $T_m(o)$, and g_0 as given by the references.

3. Transition Region at 90 km for the Tropical Atmospheric Model

We used the procedure outlined above to calculate the defining properties of all three atmospheric models. However, in the case of the tropical model, the arithmetic error between calculated and tabulated temperatures increased in magnitude and alternated in sign with increasing altitudes above 90 km. This result was caused by the poor match of data at 90 km provided by references 1 and 2. Table I shows the difference in data provided by the two references at 90 km.

Table I

DIFFERENCE IN ATMOSPHERIC DATA FOR THE ALTITUDE 90 km GIVEN BY REFERENCES 1 AND 2

	p(dynes/cm^2)	ρ(gm/cm^3)	T_m(°K)
Ref 1	1.6438	3.1700×10^{-9}	180.65
Ref 2	1.8620	3.5224×10^{-9}	184.15

Therefore, we created a transition region between 90 and 110 km to effect a smooth transition from the data in reference 2 (0 to 90 km) to the data in reference 1 (90 to 700 km). We did this by replacing the two altitude groups from 90 to 100 km and from 100 to 110 km by two groups from 90 km to an altitude, z, to be determined, and from z to 110 km.

We determined the altitude, z, in the following manner. First, the basic procedure outlined in section II-2 was used to calculate the molecular scale temperatures and gradients for the altitude groups from 0 to 90 km, using the data in reference 2. Next, we assumed that the isothermal region from 80.13 to 90 km actually extends from 80.13 km to the altitude, z; that is, $T_m(80.13) = T_m(90) = T_m(z)$ or $L(80.13 \rightarrow z) = 0$. Then we calculated the altitude, z, by iteration according to the analysis below.

Since

$$\ln\left[\frac{p(110)}{p(90)}\right] \equiv \ln\left[\frac{p(z)}{p(90)}\right]_{(L=0)} + \ln\left[\frac{p(110)}{p(z)}\right]_{(L\neq 0)} \qquad (10)$$

we have an equation that can be solved by an iterative method to give the

altitude z. The first term on the right of equation 10 becomes

$$\ln \frac{p(z)}{p(90)} = - \frac{g_o M_o a^2}{T_m(90)R} \frac{z-90}{(a+z)(a+90)} \tag{11}$$

The second term becomes

$$\ln \frac{p(110)}{p(z)} = - \frac{g_o M_o a^2}{\left[T_m(90) - (L)(a+90)\right]^2 R} \left\{ \frac{\left[T_m(90) - (L)(a+90)\right](z-90)}{(a+z)(a+90)} + L\ln\left[\frac{a+90}{a+z} \frac{T_m(90) + (L)(z-90)}{T_m(90)}\right] \right\} \tag{12}$$

The values p(90) and g_o are known from reference 2; the values p(110), a, R, and $T_m(110)$ from reference 1; and $T_m(90)$ from the calculation. Since L is a function of z

$$L = \frac{T_m(110) - T_m(90)}{110 - z} \tag{13}$$

equation 10 can be solved by iteration to find z.

After finding the altitude z, and p(z) by equation 11, we resumed the calculation of molecular scale temperatures and gradients for the altitude groups from 90 to 700 km, using the method outlined in section II-2.

Similar differences between references 3 and 1 exist at 90 km for the subarctic atmosphere. However, it was possible to generate a hydrostatically stable model without resorting to an artificial transition region. Research on the subarctic model will continue to determine whether such a region will indeed improve it. In any case, the present subarctic model is adequate for our purposes.

SECTION III

APPLICATIONS

When fitting an atmosphere into a mesh of cells for the purpose of solving the hydrodynamic equations numerically on a computer, it is necessary to define a density and specific internal energy as well as a pressure to the atmosphere contained in each cell. Therefore, for a given altitude z

$$p(z) = p\left(z_i\right)e^{-f} \tag{14}$$

where

$$f = \frac{g_o M_o a^2}{\left[T_m\left(z_i\right) - \left(L_i\right)\left(a+z_i\right)\right]^2 R} \left\{ \frac{\left[T_m\left(z_i\right) - \left(L_i\right)\left(a+z_i\right)\right]\left(z-z_i\right)}{(a+z)\left(a+z_i\right)} + L_i \ln\left[\frac{a+z_i}{a+z}\, \frac{T_m\left(z_i\right) + \left(L_i\right)\left(z-z_i\right)}{T_m\left(z_i\right)}\right]\right\} \tag{15}$$

and

$$z_i \le z < z_{i+1}$$

By the ideal gas law, the density at altitude z is

$$\rho(z) = \frac{p(z)}{p(o)}\,\frac{T_m(o)}{T_m(z)}\,\rho(o) \tag{16}$$

where $p(o)$, $\rho(o)$, and $T_m(o)$ are the sea level values for pressure, density, and molecular scale temperature, respectively.

To obtain the specific internal energy, $I(z)$, we require the equation of state for air which will be used in the calculation (e.g., the Doan-Nickel Equation of State for Air, reference 4) to return the same pressure as the

atmospheric model. This involves a solution by iteration which proceeds as follows. Guess a value for γ. Then, by the ideal gas law,

$$I(z) = \frac{p(z)}{(\gamma-1)\rho(z)} \tag{17}$$

where $p(z)$ and $\rho(z)$ are the values of pressure and density at altitude, z, as defined by the atmospheric model. Then these values for $I(z)$ and $\rho(z)$ are entered into the equation of state for air which returns a value for $\left(\gamma_{effective} - 1\right)$. Then, by the ideal gas law, the pressure is

$$p(z) = \left(\gamma_{effective} - 1\right)\rho(z)I(z) \tag{18}$$

The iteration continues until that combination of γ and $I(z)$ is found, which will make the equation of state deliver the same pressure as the atmospheric model. See appendix.

SECTION IV

RESULTS

The annual mean temperate atmospheric model, the annual mean tropical atmospheric model, and the summer subarctic atmospheric model are given in tables II, III, and IV, respectively. Tables V, VI, and VII show the difference between the molecular scale temperatures at the base altitude of each altitude group as listed in references 1, 2, and 3 and those calculated for the atmospheric models.

The value for the altitude, z, used in the transition region of the tropical atmosphere is 97.84061 km; that is, the two altitude groups making up the transition region extend from 90 to 97.84061 km and from 97.84061 to 110 km.

The appendix contains the models in subroutine form.

Table II

DEFINING PROPERTIES OF THE ANNUAL MEAN TEMPERATE ATMOSPHERE
(45° N latitude)

Altitude z(cm)	Pressure (dynes/cm^2)	Molecular scale temperature gradient L (°K/cm)	Molecular scale temperature T_m (°K)
0.0000	1.01325×10^{6}	$-6.49291767 \times 10^{-5}$	288.150000
1.1019×10^{6}	2.26320×10^{5}	$9.28049177 \times 10^{-8}$	216.604540
2.0063×10^{6}	5.47487×10^{4}	$9.86254816 \times 10^{-6}$	216.688473
3.2162×10^{6}	8.68014×10^{3}	$2.77080370 \times 10^{-5}$	228.621170
4.7350×10^{6}	1.10905×10^{3}	$-1.72246873 \times 10^{-7}$	270.704137
5.2429×10^{6}	5.90005×10^{2}	$-1.95999298 \times 10^{-5}$	270.616652
6.1591×10^{6}	1.82099×10^{2}	$-3.91697376 \times 10^{-5}$	252.659197
7.9994×10^{6}	1.03770×10^{1}	$1.60823156 \times 10^{-7}$	180.575129
9.0000×10^{6}	1.64380×10^{0}	2.9816673[illegible] $\times 10^{-5}$	180.736048
1.0000×10^{7}	3.007[illegible] $\times 10^{-1}$	$5.02020153 \times 10^{-5}$	210.552722
1.1000×10^{7}	7.35440×10^{-2}	$9.97762308 \times 10^{-5}$	260.754737
1.2000×10^{7}	2.52170×10^{-2}	$2.00108806 \times 10^{-4}$	360.530968
1.5000×10^{7}	5.06170×10^{-3}	$1.49589024 \times 10^{-4}$	960.857386
1.6000×10^{7}	3.694[illegible]0 $\times 10^{-3}$	$1.00407491 \times 10^{-4}$	1110.446410
1.7000×10^{7}	2.79260×10^{-3}	$6.97598503 \times 10^{-5}$	1210.853900
1.9000×10^{7}	1.68520×10^{-3}	$5.01601097 \times 10^{-5}$	1350.373600
2.3000×10^{7}	6.96040×10^{-4}	$3.98897144 \times 10^{-5}$	1551.014040
3.0000×10^{7}	1.88380×10^{-4}	$3.31099390 \times 10^{-5}$	1830.242040
4.0000×10^{7}	4.03040×10^{-5}	$2.58868496 \times 10^{-5}$	2161.341430
5.0000×10^{7}	1.09570×10^{-5}	$1.71252931 \times 10^{-5}$	2420.209930
6.0000×10^{7}	3.45020×10^{-6}	$1.09162418 \times 10^{-5}$	2591.462860
7.0000×10^{7}	1.19180×10^{-6}	...	2700.625280

$a = 6.35670 \times 10^{8}$ cm

$g = 9.80665 \times 10^{2}$ cm/sec^2

$R = 8.31440 \times 10^{7}$ erg/mole/deg K.

$M = 2.89644 \times 10^{1}$ gm/mole

Table III

DEFINING PROPERTIES OF THE ANNUAL MEAN TROPICAL ATMOSPHERE
(15° N latitude)

Altitude z(cm)	Pressure (dynes/cm^2)	Molecular scale temperature gradient L (°K/cm)	Molecular scale temperature T_m (°K)
0.000000	1.013250×10^6	$-3.69962272 \times 10^{-5}$	299.650000
2.254000×10^5	7.813300×10^5	$-1.29419605 \times 10^{-4}$	291.311050
2.505000×10^5	7.586100×10^5	$-6.75109163 \times 10^{-5}$	288.062618
1.657000×10^6	1.013700×10^5	$4.11001824 \times 10^{-5}$	193.108515
2.211300×10^6	4.043000×10^4	$2.14811736 \times 10^{-5}$	215.890346
4.743200×10^6	1.188600×10^3	$4.20150001 \times 10^{-7}$	270.278529
5.149800×10^6	7.175600×10^2	$-1.93134924 \times 10^{-5}$	270.449362
5.965200×10^6	2.535100×10^2	$-3.43722820 \times 10^{-5}$	254.701140
8.013000×10^6	1.100100×10^1	$1.98418410 \times 10^{-7}$	184.313581
9.000000×10^6	1.862000×10^0	$-9.16634469 \times 10^{-7}$	184.509420
9.784061×10^6	4.549471×10^{-1}	$6.35832871 \times 10^{-5}$	183.790723
1.100000×10^7	7.354400×10^{-2}	$9.74012527 \times 10^{-5}$	261.104121
1.200000×10^7	2.521700×10^{-2}	$2.00803436 \times 10^{-4}$	358.505374
1.500000×10^7	5.061700×10^{-3}	$1.44417115 \times 10^{-4}$	960.915680
1.600000×10^7	3.694300×10^{-3}	$1.05385251 \times 10^{-4}$	1105.332790
1.700000×10^7	2.792600×10^{-3}	$6.68110562 \times 10^{-5}$	1210.718050
1.900000×10^7	1.685200×10^{-3}	$5.15616802 \times 10^{-5}$	1344.340160
2.300000×10^7	6.960400×10^{-4}	$3.88307486 \times 10^{-5}$	1550.586880
3.000000×10^7	1.883800×10^{-4}	$3.37947184 \times 10^{-5}$	1822.402120
4.000000×10^7	4.030400×10^{-5}	$2.49828094 \times 10^{-5}$	2160.349300
5.000000×10^7	1.095700×10^{-5}	$1.79900870 \times 10^{-5}$	2410.177400
6.000000×10^7	3.450200×10^{-6}	$9.94365149 \times 10^{-6}$	2590.078270
7.000000×10^7	1.191800×10^{-6}	...	2689.514780

$a = 6.35670 \times 10^8$ cm

$g_0 = 9.78381 \times 10^2$ cm/sec^2

$R = 8.31440 \times 10^7$ erg/mole/deg K.

$M = 2.99644 \times 10^1$ gm/mole

Table IV

DEFINING PROPERTIES OF THE SUMMER SUBARCTIC ATMOSPHERE
(60° N latitude)

Altitude z(cm)	Pressure (dynes/cm^2)	Molecular scale temperature gradient L (°K/cm)	Molecular scale temperature T_m (°K)
0.0000	1.0100×10^{6}	$-5.15226593 \times 10^{-5}$	287.150000
4.9980×10^{5}	5.4154×10^{5}	$-7.45507917 \times 10^{-5}$	261.398975
1.0003×10^{6}	2.6758×10^{5}	$1.75752872 \times 10^{-6}$	224.086304
2.3054×10^{6}	3.7248×10^{4}	$1.21402825 \times 10^{-5}$	226.380054
3.2121×10^{6}	9.8883×10^{3}	$3.20422887 \times 10^{-5}$	237.387649
4.3237×10^{6}	2.2624×10^{3}	$5.54484005 \times 10^{-6}$	273.005857
4.8303×10^{6}	1.2140×10^{3}	$5.03741727 \times 10^{-6}$	275.814873
5.3376×10^{6}	6.5545×10^{2}	$-2.35321674 \times 10^{-5}$	278.370354
5.9476×10^{6}	3.0773×10^{2}	$-4.51273514 \times 10^{-5}$	264.015732
7.9890×10^{6}	1.2772×10^{1}	$-1.48868732 \times 10^{-6}$	171.892757
9.0000×10^{6}	1.7851×10^{0}	$3.31280965 \times 10^{-5}$	170.387694
1.0000×10^{7}	3.0075×10^{-1}	$6.61955253 \times 10^{-5}$	203.515791
1.1000×10^{7}	7.3544×10^{-2}	$8.07805343 \times 10^{-5}$	269.711316
1.2000×10^{7}	2.5217×10^{-2}	$2.10740515 \times 10^{-4}$	350.491850
1.5000×10^{7}	5.0617×10^{-3}	$1.06709040 \times 10^{-4}$	982.713396
1.6000×10^{7}	3.6943×10^{-3}	$1.47081912 \times 10^{-4}$	1089.422440
1.7000×10^{7}	2.7926×10^{-3}	$4.49679987 \times 10^{-5}$	1236.504250
1.9000×10^{7}	1.6852×10^{-3}	$6.37859718 \times 10^{-5}$	1326.440340
2.3000×10^{7}	6.9604×10^{-4}	$3.13145277 \times 10^{-5}$	1581.584230
3.0000×10^{7}	1.8838×10^{-4}	$3.99574107 \times 10^{-5}$	1800.785930
4.0000×10^{7}	4.0304×10^{-5}	$1.53856612 \times 10^{-5}$	2200.360030
5.0000×10^{7}	1.0957×10^{-5}	$2.32271247 \times 10^{-6}$	2384.216670
6.0000×10^{7}	3.4502×10^{-6}	$2.47207416 \times 10^{-6}$	2636.487910
7.0000×10^{7}	1.1918×10^{-6}	...	2661.208650

$a = 6.35670 \times 10^{8}$ cm

$g_0 = 9.81911 \times 10^{2}$ cm/sec^2

$R = 8.31440 \times 10^{7}$ erg/mole/deg K

$M = 2.89644 \times 10^{1}$ gm/mole

Table V

DIFFERENCES BETWEEN CALCULATED AND TABULATED MOLECULAR SCALE TEMPERATURES IN THE TEMPERATE ATMOSPHERIC MODEL

Altitude (cm)	Tabulated temperatures (°K)	Calculated temperatures (°K)	% Difference
0.0000	288.15	288.15	0.000
1.1019×10^6	216.65	216.60	-0.021
2.0063×10^6	216.65	216.69	0.018
3.2162×10^6	228.65	228.62	-0.013
4.7350×10^6	270.65	270.70	0.020
5.2429×10^6	270.65	270.62	-0.012
6.1591×10^6	252.65	252.66	0.004
7.9994×10^6	180.65	180.58	-0.041
9.0000×10^6	180.65	180.74	0.048
1.0000×10^7	210.65	210.55	-0.046
1.1000×10^7	260.65	260.75	0.040
1.2000×10^7	360.65	360.53	-0.033
1.5000×10^7	960.65	960.86	0.022
1.6000×10^7	1110.65	1110.45	-0.018
1.7000×10^7	1210.65	1210.85	0.017
1.9000×10^7	1350.65	1350.37	-0.020
2.3000×10^7	1550.65	1551.01	0.023
3.0000×10^7	1830.65	1830.24	-0.022
4.0000×10^7	2160.65	2161.36	0.032
5.0000×10^7	2420.65	2420.21	-0.018
6.0000×10^7	2590.65	2591.46	0.031
7.0000×10^7	2700.65	2700.63	-0.001

Table VI

DIFFERENCES BETWEEN CALCULATED AND TABULATED MOLECULAR SCALE TEMPERATURES IN THE TROPICAL ATMOSPHERIC MODEL

Altitude (cm)	Tabulated temperatures (°K)	Calculated temperatures (°K)	% Difference
0.000000	299.65	299.65	0.000
2.254000×10^5	286.15	291.31	1.804
2.505000×10^5	286.95	288.06	0.388
1.657000×10^6	193.15	193.11	-0.021
$2.211[illegible] \times 10^6$	215.15	215.89	0.344
4.743200×10^6	270.15	270.28	0.048
5.198000×10^6	270.15	270.45	0.111
5.965200×10^6	254.15	254.70	0.217
8.013000×10^6	184.15	184.31	0.089
9.000000×10^6	184.15	184.51	0.925
9.784061×10^7	184.15	183.79	-0.191
1.100000×10^7	260.65	261.10	0.174
1.200000×10^7	360.65	358.51	-0.595
1.500000×10^7	960.65	960.92	0.028
1.600000×10^7	1110.65	1105.33	-0.478
1.700000×10^7	1210.65	1210.72	0.006
1.900000×10^7	1350.65	1344.34	-0.467
2.300000×10^7	1550.65	1550.59	-0.004
3.000000×10^7	1830.65	1822.40	-0.451
4.000000×10^7	2160.65	2160.35	-0.014
5.000000×10^7	2420.65	2410.18	-0.432
6.000000×10^7	2590.65	2590.08	-0.022
7.000000×10^7	2700.65	2689.51	0.412

Table VII

DIFFERENCES BETWEEN CALCULATED AND TABULATED MOLECULAR SCALE TEMPERATURES IN THE SUBARCTIC ATMOSPHERIC MODEL

Altitude (cm)	Tabulated temperatures (°K)	Calculated temperatures (°K)	% Difference
0.0000	287.15	287.15	0.000
4.9980×10^5	260.15	261.40	0.480
1.0003×10^6	225.15	224.09	-0.472
2.3054×10^6	225.15	226.38	0.546
3.2121×10^6	238.65	237.39	-0.529
4.3237×10^6	271.65	273.00	0.499
4.8303×10^6	277.15	275.81	-0.482
5.3376×10^6	277.15	278.37	0.440
5.9476×10^6	265.15	264.02	-0.428
7.9890×10^6	171.15	171.90	0.434
9.0000×10^6	171.15	170.39	-0.445
1.0000×10^7	210.65	203.52	-3.387
1.1000×10^7	260.65	269.71	3.476
1.2000×10^7	360.65	350.49	-2.817
1.5000×10^7	960.65	982.71	2.297
1.6000×10^7	1110.65	1089.42	-1.911
1.7000×10^7	1210.65	1236.50	2.136
1.9000×10^7	1350.65	1326.44	-1.792
2.3000×10^7	1550.65	1591.58	1.9[illegible]
3.0000×10^7	1830.65	1800.79	-1.631
4.0000×10^7	2160.65	2200.36	1.837
5.0000×10^7	2420.65	2384.22	-1.505
6.0000×10^7	2590.65	2636.49	1.769
7.0000×10^7	2700.65	2661.21	-1.460

SECTION V

CONCLUSIONS

The atmospheric models are derived such that the hydrostatic equation is satisfied.

$$\frac{d\left[p(z)\right]}{dz} = -\rho(z)g(z)$$

In general, hydrodynamic computer codes calculate dp/dz, the pressure gradient, by finite difference methods which replace dp/dz by $\Delta p/\Delta z$. This approximation results in unwanted accelerations which can be written as

$$a = \frac{\left(\frac{dp}{dz}\right)_z - \left(\frac{\Delta p}{\Delta z}\right)_z}{\rho(z)}$$

For example, given a code in which

$$\left(\frac{\Delta p}{\Delta z}\right)_z = \frac{p(z+\Delta z) - p(z-\Delta z)}{2\Delta z}$$

and assuming a scale height, H, constant over the interval, Δz, so that the pressure, p(z), can be written

$$p(z) = p\left(z_o\right)e^{-\frac{z-z_o}{H}}$$

one can expand $(\Delta p/\Delta z)$ in powers of $\Delta z/H$ so that to second order, the net accelerations are

$$a = \frac{1}{6}\, g(z) \left(\frac{\Delta z}{H}\right)^2$$

For all practical purposes, this acceleration is negligible until the zone size, Δz, becomes greater than a tenth of a kilometer. For example, at an altitude of 250 km, the scale height, H, is about 5 kilometers and g about 900 cm/sec^2. At 250 km, the scale height has its smallest value in the atmosphere so this example will give a measure of the worst acceleration. If Δz is 0.1 km, the above relationship gives an acceleration of 0.05 cm/sec^2. If Δz is 1 km, the acceleration is 5.0 cm/sec^2.

The pressures and densities given by the atmospheric models agree with tabulated values in references 1 and 2 to one part in 10^8 and temperatures to one part in 100.

APPENDIX

SUBROUTINE ATMOS

The FORTRAN subroutine appearing in this appendix is the annual mean temperate atmospheric model. By inputting an altitude, TTY, the subroutine will return

WSP	the pressure at that altitude
WSR	the density at that altitude
WST	the temperature at that altitude
WSI	the specific internal energy at that altitude
WSU	the radial velocity = 0
WSV	the axial velocity = 0
GMONE	$\gamma - 1.$

For the annual mean tropical atmospheric model or the summer subarctic atmospheric model, replace the data in the TABZ, TABL, TABT, and TABP blocks with the data appearing in tables III or IV as appropriate.

```
      SUBROUTINE ATMOS(Z,Y,WSP,WSR,WST,WSI,WSU,WSV,GMONE)
      DIMENSION TABZ(22), TABL(22), TABT(22), TABP(22)
C
C           CALCULATE ATMOSPHERE
C
C     A=RADIUS OF THE EARTH IN CM.
C     G=ACCELERATION DUE TO GRAVITY IN CM./SEC./SEC.
C      =980.665 FOR THE TEMPERATE ATMOSPHERE
C      =978.381 FOR THE TROPICAL ATMOSPHERE
C      =981.911 FOR THE SUBARCTIC ATMOSPHERE
C     R=GAS CONSTANT IN ERGS/MOLE/DEG.
C     W=MOLECULAR WEIGHT OF AIR
C
      A=6.3567E+08
      G=9.80665E+02
      R=8.3144E+07
      W=2.89644E+01
C
C     THE FOLLOWING DATA IN TABZ, TABL, TABT, AND TABP ARE FROM TABLE 2
C     FOR THE TEMPERATE ATMOSPHERE. FOR THE TROPICAL OR SUBARCTIC ATMO-
C     SPHERES SUBSTITUTE THE DATA FROM TABLES 3 AND 4 RESPECTIVELY.
C
C     TABZ IS THE BASE ALTITUDE OF EACH ALTITUDE GROUP IN CM.
      TABZ( 1)=0.
      TABZ( 2)=1.1019E+06
      TABZ( 3)=2.0063E+06
      TABZ( 4)=3.2162E+06
      TABZ( 5)=4.7350E+06
      TABZ( 6)=5.2429E+06
      TABZ( 7)=6.1591E+06
      TABZ( 8)=7.9994E+06
      TABZ( 9)=9.0E+06
      TABZ(10)=10.0E+06
      TABZ(11)=11.0E+06
      TABZ(12)=12.0E+06
      TABZ(13)=15.0E+06
      TABZ(14)=16.0E+06
      TABZ(15)=17.0E+06
      TABZ(16)=19.0E+06
      TABZ(17)=23.0E+06
      TABZ(18)=30.0E+06
      TABZ(19)=40.0E+06
      TABZ(20)=50.0E+06
      TABZ(21)=60.0E+06
      TABZ(22)=70.0E+06
C     TABL IS THE MOLECULAR SCALE TEMPERATURE GRADIENT OF EACH ALTITUDE
C     GROUP IN DEG./CM.
      TABL( 1)=-6.40291767E-05
      TABL( 2)= 9.28049177E-08
      TABL( 3)= 9.86254816E-06
      TABL( 4)= 2.77080370E-05
      TABL( 5)=-1.72246873E-07
      TABL( 6)=-1.95099298E-05
      TABL( 7)=-2.91697376E-05
      TABL( 8)= 1.60823156E-07
```

```
      TABL( 9)= 2.98166706E-05
      TABL(10)= 5.02020193E-05
      TABL(11)= 9.97762308E-05
      TABL(12)= 2.00108806E-04
      TABL(13)= 1.49589024E-04
      TABL(14)= 1.00407491E-04
      TABL(15)= 6.97598503E-05
      TABL(16)= 5.01601097E-05
      TABL(17)= 3.98997144E-05
      TABL(18)= 3.31099390E-05
      TABL(19)= 2.58858496E-05
      TABL(20)= 1.71252921E-05
      TABL(21)= 1.09162418E-05
C     TABT IS THE MOLECULAR SCALE TEMPERATURE CORRESPONDING TO THE TABZ
C     IN DEGREES KELVIN
      TABT( 1)= 2.88150000E+02
      TABT( 2)= 2.16604540E+02
      TABT( 3)= 2.16688473E+02
      TABT( 4)= 2.28621170E+02
      TABT( 5)= 2.70704137E+02
      TABT( 6)= 2.70616652E+02
      TABT( 7)= 2.52659197E+02
      TABT( 8)= 1.80575129E+02
      TABT( 9)= 1.90736048E+02
      TABT(10)= 2.10552722E+02
      TABT(11)= 2.60754737E+02
      TABT(12)= 3.60530968E+02
      TABT(13)= 9.60857386E+02
      TABT(14)= 1.11044641E+03
      TABT(15)= 1.21085390E+03
      TABT(16)= 1.35037360E+03
      TABT(17)= 1.55101404E+03
      TABT(18)= 1.83024204E+03
      TABT(19)= 2.16134143E+03
      TABT(20)= 2.42020993E+03
      TABT(21)= 2.59146286E+03
      TABT(22)= 2.70062528E+03
C     TABP IS THE PRESSURE CORRESPONDING TO THE TABZ IN DYNES/CM./CM.
      TABP( 1)=1.01325E+06
      TABP( 2)=2.26320E+05
      TABP( 3)=5.47487E+04
      TABP( 4)=8.68014E+03
      TABP( 5)=1.10905E+03
      TABP( 6)=5.90005E+02
      TABP( 7)=1.82099E+02
      TABP( 8)=1.0377E+01
      TABP( 9)=1.6438E+00
      TABP(10)=3.0075E-01
      TABP(11)=7.3544E-02
      TABP(12)=2.5217E-02
      TABP(13)=5.0617E-03
      TABP(14)=3.6943E-03
      TABP(15)=2.7926E-03
      TABP(16)=1.6852E-03
      TABP(17)=6.9604E-04
```

```
      TABP(18)=1.8838E-04
      TABP(19)=4.0304E-05
      TABP(20)=1.0957E-05
      TABP(21)=3.4502E-06
      TABP(22)=1.1918E-06
C
      RHOZ=W*TABP(1)/(TABT(1)*R)
      EZ=1.E+10
C
C     DO 80 JAT=1,21    FOR THE TEMPERATE ATMOSPHERE
C     DO 80 JAT=1,22    FOR THE TROPICAL ATMOSPHERE
C     DO 80 JAT=1,23    FOR THE SUBARCTIC ATMOSPHERE
C
      DO 80  JAT=1,21
      IF(TTY-TABZ(JAT))81,82,80
80    CONTINUE
C     JAT=21 FOR THE TEMPERATE ATMOSPHERE
C     JAT=22 FOR THE TROPICAL ATMOSPHERE
C     JAT=23 FOR THE SUBARCTIC ATMOSPHERE
      JAT=21
      GO TO 82
81    JAT=JAT-1
   82 CONS=A*A*G*W/R
      DUM2=(TTY-TABZ(JAT))/((A+TTY)*(A+TABZ(JAT)))
      DUM3=(A+TABZ(JAT))/(A+TTY)
      VAR1=TABT(JAT)-TABL(JAT)*(A+TABZ(JAT))
      VAR2=(TABT(JAT)+TABL(JAT)*(TTY-TABZ(JAT)))/TABT(JAT)
      FS=CONS/(VAR1*VAR1)*(VAR1*DUM2+TABL(JAT)*LOGF(DUM3*VAR2))
      WSP=TABP(JAT)*EXPF(-FS)
      WST=TABT(JAT)+TABL(JAT)*(TTY-TABZ(JAT))
      WSR=WSP*TABT(1)*RHOZ    /(WST*TABP(1))
C
C    CALCULATE THE INTERNAL ENERGY
C
      FSO=0.
      GAM1=.5
      DGM=-0.001
900   RHO=WSR
      E=WSP/(GAM1*WSR)
      ENY=E
C
C     DOAN-NICKEL SEMI-PHYSICAL FIT TO THE EQUATION OF STATE OF AIR
C
C        TEMPERATURES FROM .025 TO 1.5 ELECTRON VOLTS
C        DENSITIES FROM 10**2 TO 10**(-7) NORMAL DENSITY
C        PRESSURE = (GAMMA-1.)*RHO*E, WHERE GAMMA IS A FUNCTION OF
C         DENSITY AND ENERGY
C     RHO = MATERIAL DENSITY
C     RHOZ = 1.2935E MEGAGRAMS/CUBIC KILOMETER, IN THE UNITS OF THE
C         PROBLEM
C     E = ENERGY/MASS
C     EZ = 1 JERK/MEGAGRAM, IN THE UNITS OF THE PROBLEM
C     GMONE = GAMMA MINUS ONE
C     MAKE E POSITIVE IF NEGATIVE, AND CONVERT TO JERKS/MEGAGRAM
102   E=ABSF(E)/EZ
```

```
      POWER=-ALOG(RHO/RHOZ)/2.3025851
C     THE ENERGY AT WHICH OXYGEN AND NITROGEN DISSOCIATE IS A
C     FUNCTION OF DENSITY
      E1=(8.5-E)/.975
C     THE FERMI-DIRAC FUNCTION IS ONLY COMPUTED WITHIN 5.*DELTA E OF
C     EACH TRANSITION. OTHERWISE IT IS ONE OR ZERO
      IF (ABSF(E1)-5.) 106,103,103
 103  IF (E1) 105,105,104
 104  FO=EXPF(-E/4.46)
      FON=0.
      WS=1.
      GO TO 107
 105  FO=0.
      FON=EXPF(-E/6.63)
      WS=0.
      GO TO 107
 106  DE1=.075*(RHO/RHOZ)**.05
      EE1=8.5-.357*POWER
      E1=(EE1-E)/DE1
      WS=1./(EXPF(-E1)+1.)
      FO=EXPF(-E/4.46)*WS
      FON=EXPF(-E/6.63)*(1.-WS)
C     THE DENSITY DEPENDENCE ONLY OCCURS ABOVE E=1., AND IT IS OF
C     THE FORM (RHO/RHOZ)**(CONSTANT*LOG(E)).  THE CONSTANT
C     MAKES A TRANSITION FROM .048 TO .029 AS THE OXYGEN DISSOCIATES
C     AND THE DENSITY SPREAD BECOMES CONSTANT BEYOND THE THIRD PEAK
 107  IF(E-1.) 108,108,109
 108  BETA=0.
      GO TO 110
 109  BETA=(.048*WS+.032*(1.-WS))*ALOG(E)/2.3025851
 110  E2=(E-40.)/3.
      IF (ABSF(E2)-5.) 114,111,111
 111  IF (E2) 112,112,113
 112  FN=0.
      WS=0.
      GO TO 115
 113  FN=EXPF(-E/25.5)
      WS=1.
      GO TO 115
 114  DE2=4.*(RHO/RHOZ)**.085
      EE2=45.*(RHO/RHOZ)**.0157
      E2=(E-EE2)/DE2
      WS=1./(EXPF(-E2)+1.)
      FN=EXPF(-F/25.5)*WS
 115  E3=(E-160.)/6.
      BETA=BETA*(1.-WS)+.045*WS
      IF (E3+5.) 116,117,117
 116  FE=0.
      GO TO 118
 117  FE=1./(EXPF(-E3)+1.)
 118  RHOFAC=(RHO/RHOZ)**BETA
      GMONE=(.161+.255*FO+.280*FON+.137*FN+.050*FE)*RHOFAC
      P=GMONE*ENY*RHO
      FS=P-WSP
      IF(FS) 999,1004,999
```

```
999    IF(FS*FSO)1001,1000,1000
1000   GAM1=GAM1+DGM
       FSO=FS
       GO TO 1002
1001   GAM1=GAM1-DGM
       DGM=DGM/100.
       GAM1=GAM1+DGM
1002   IF(GAM1)1004,1003,1003
 1003 IF(ABSF(DGM)-1.0E-10)1004,900,900
1004   WSI=  P/(GMONF*WSR)
       WSU=0.
       WSV=0.
       RETURN
       END
```

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A hydrostatically stable atmospheric model is necessary to perform theoretical calculations of hydrodynamic motion in the atmosphere, on a digital computer. This report presents three such models developed at the Air Force Weapons Laboratory for use in its hydrodynamic computer codes. One is for the annual mean temperate atmosphere (45° N latitude); one for the annual mean tropical atmosphere (15° N latitude); and one for the summer subarctic atmosphere (60° N latitude). The models are presented herein in tabular form and as FORTRAN subroutines which could be placed directly into any hydrodynamic computer code. For a given altitude (cm), the subroutines return a pressure (dynes/cm^2), density (gms/cm^3), specific internal energy (ergs/gm), temperature (°K), and $(\gamma - 1)$. The pressures and densities agree with tabulated values to at least 1 part in 10^8 and temperatures to at least 1 part in 100. The atmospheres experience an acceleration of no more than 3 parts in 10^3 in a first order finite difference scheme with a zone size of 1 kilometer, the worst case.

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